Elongation of Contact Length on the Line of Action in Roll Forming of Gears

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The elongation of contact length on the line of action is considered with particular reference for roll forming of gears, and for dynamic behavior of the tooth in meshing. However there is no paper that discuss the elongation of contact length in the load meshing of gears. Based on our investigation, the contact length on the line of action elongates more than the kinematically calculated value. In rolling, as the tool approaches the workpiece, the center distance of the gears decreases by a small amount. But, the elongation of contact length is sensitive. Therefore, the contact point on the line of action is difficult to be determined, which complicates the tooth analysis. In this study, the exact relation between the elongation of contact length and the tooth space over the recess or before the approach are revealed by experiments and kinematic theory. This analytical result applies not only for rolling, but also for the single flank meshing which is done under constant center distance.

Key Words : Elongation, Tooth Tip, Corner Contact, Line of Action, Tooth Error, Deflection

1. Introduction

Gears are some of the most frequently used power transmission companents in cars, airplanes, and industrial machines. Because of greater industrial development, there is an increasing demand for more efficient, high precision gears that are stronger and more powerful, yet smaller and lighter. Various research efforts have been advanced to design gears that meet these increasing demands and overcome the limitations facing the gear-manufacturing industry (Aida et al., 1967; Lyu et al., 1994, 1996, 1998).

Up to date, it has not been recognized that the contact length on the line of action elongates in the load meshing of gears. In the research of the dynamic meshing of gears, Akiyama considers the contact at the tooth tip (Akiyama et al., 1968). Also, Seager discusses the contact during scoring (Seager, 1976).

We have considered the problem in the case of tooth analysis of rolling gears (Uematsu, 1988). We proved that one do not disregard the elongation of the contact length in order to carry out the analysis accurately. These experiments were carried out using a rack type tool without profile modification. In the analysis of the results, by considering that the pinion tooth contacts with the tool on both sides of the tooth, the total length of line of action was calculated. However, the calculated value of the contact length did not completely agree with the experimental value acquired with strain gauge installed at the root of the tool tooth. The contact was considered at the tooth tip, as proposed by (Akiyama et al., 1968) and (Seager, 1976). But the experimental values did not agree with the calculated ones. For ex-

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ample, the calculated value and the experimental value were 38.6 mm and 42.5 mm, respectively. However, the normal pitch required from the experiment agreed with the calculated value.

Because the contact occurs on both sides of the tooth during rolling, the problem is more complicated for the calculation of the contact phase. Therefore, the calculation method was reexamined. In the meantime, the tool utilized in the experiments was made using milling cutter No. 1. As the tool has 135 teeth, the tooth profile is considerably modified on the addendum. The experimental value of total contact length on the line of action and normal pitch agreed with the calculated values.

As a result, when the rolling is realized by using a non-modifying tool and normal tooth profile of the pinion, one part of the contact length elongates. But the normal pitch does not. In order to specify which part of the contact length elongates, the behavior of the rolled tooth was also observed. It was proven that the contact length elongated before the beginning contact and slightly after the final contact.

In this research, the phenomenon in which the contact length elongates in the loading is handled not only as a rolling problem but also as a problem of the single flank contact. Theoretically, it was clarified by considering the dimensions of the gear, deflection of the tooth, tooth profile error, and tool approach to the pinion on the center distance. According to our method, the elongation of contact length in the case of a single flank can be explained as a case in which the tool does not approach the pinion axially.

2. Geometry of Double Flank Contact

A pair of gears contacting in double flank is shown in Fig. 1(a). The pair is rack and pinion. The transverse direction of the rack is from right to left, and the direction of rotation on the pinion is clockwise. The tooth number of rack is given as T_1, T_2 in the transverse direction. The line of action in the drive side of the rack is shown as A_dPR_d, and the back as A_fPR_f. A_d and A_f show the beginning contact points on each line of action, and R_d and R_f show the final contact points. P shows the pitch point.

To define the phase lag between the drive side and the driven side, let's watch for the contact process in some teeth. The drive side of rack contacts the pinion from point A_d, and the other side contacts from A_f. This phase is shown as δ Lp on the line of action.

In Fig. 1(b), when the follower side of the rack is located at the pitch point, the drive side of rack is located at R on the line of action. Therefore, the drive side of rack passes through the pitch point, and goes through the distance PR on the line of action. Phase δ Lp is shown in Fig. 1(c). Based on this figure, we can derive the following equation:

$$\Delta Lp = A_dP + PR - A_fP = R_g(\tan\alpha_k - \tan\alpha_c) + m(\pi/2 - 2x\tan\alpha_c) \cdot \cos\alpha_c - m(k-x)/\sin\alpha_c \quad (1)$$

In Fig. 1(b), km is the addendum height of the rack, and k is the coefficient of the addendum height.

The state of contact on rack and pinion is shown on the mating chart. In this figure, the geometrical contact length of every tooth is shown as A_dR_d and A_fR_f, and they are ordered in tooth number, and described and shifted by the normal pitch. In other words, the number of the contacting segment is assigned to the tooth number of the tool. The number of the tooth is assigned to be T_2, and T_3, $\cdots$, T_n, number of the tooth flank to be D_2, D_3, $\cdots$, D_n, F_2, and F_3, $\cdots$, F_n.

3. Measurement of Contact Length

The module of rack used in the experiments is 5 and the pressure angle is 20°. The tool is rounded at the tooth tip and the height of addendum is 4.3 mm. The test gear has 22 teeth and A.M. factors are 0 and 0.52. The contact length in rolling is obtained as an output of the strain gauge installed at the tooth root of the tool.

The detection method of the deflection of the pinion tooth is shown in Fig. 2. The steel rod is installed at the addendum in cross section, and its diameter is 2 mm. The circumferential displace-

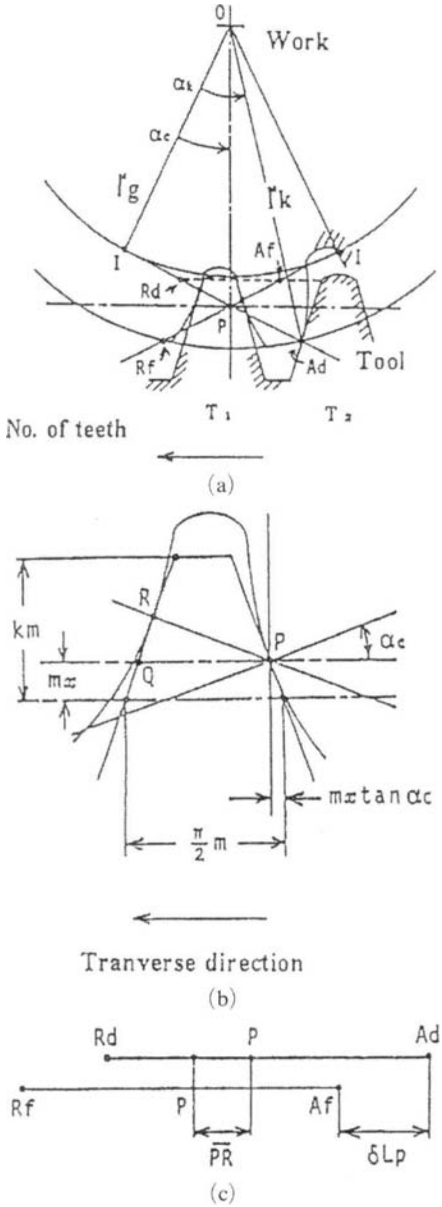

Fig. 1 Contact length and line of action for rack and pinion

ment of the pin was made to be the deflection of the tooth. The displacement of the pin is measured by the strain gauge installed in a cantilever beam on the cross section. This beam was made of phosphor bronze board of 5 mm width and 0.1 mm thickness.

The measured result of the contact length is shown in Fig. 3. The A.M. factor of the gear is

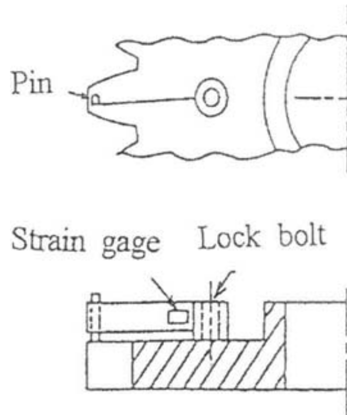

Fig. 2 Tooth deflection sensor

0.52. It is the strain of the tool at the tooth root and the displacement of the pinion at the tooth tip. These were shown on the mating diagram. The geometrical contact length was shown in the solid line A_dR_d and A_fR_f. When the tool approaches the pinion, and the tooth tip of pinion contacts with the tool, it is shown by the broken lines A_dA_d', R_dR_d', A_fA_f' and R_fR_f'.

The beginning and final contact points are examined. The pinion tooth to detect the deflection contacts with profile D_3 and F_2 of the tool. The beginning contact point on the profile D_3 of the tool is $A_d'W$ and the final contact point in the profile F_2 is $R_f'W$. Next, we observe on the peak position, the beginning contact point, and the final contact point on the strain curve of tooth root at T_2 and T_3. The peak strain at the tooth root appears near ΔC, ΔE in the lower side of the chart. The distance between the peaks on the line of action is precisely a normal pitch. The beginning contact point and the final contact point on the tooth of the tool are confirmed from the displacement of the tooth tip on the pinion tooth.

The beginning contact point $A_d'3$ on tooth T_3 corresponds to point $A_d'W$ on tooth displacement of the pinion. The final contact point $R_f'_2$ on tooth T_2 of the tool corresponds to point $R_f'W$ on tooth displacement of the pinion. The final contact point $R_f'_3$ on tooth T_3 of the tool separates from $R_f'W$ as much as the normal pitch. As a result, the total contact length for the tooth be-

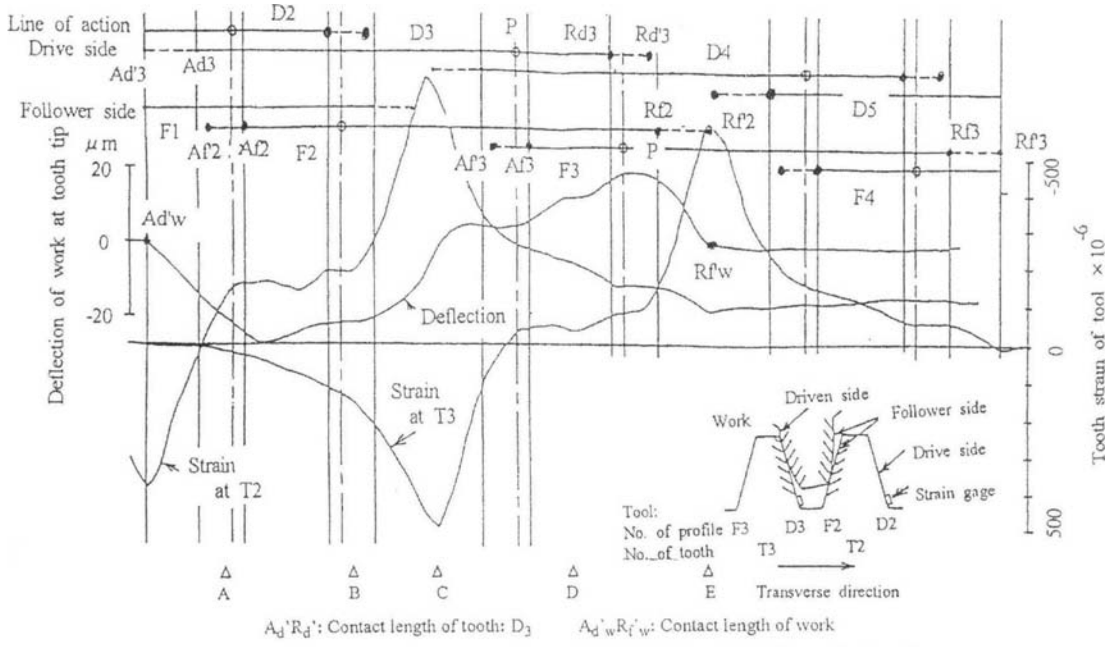

Fig. 3 Tooth stress and deflection in rolling

comes $A_d{}'_3R_f{}'_3$, and the length is 42.5 mm.

At the final contact point Rd on the drive side and the beginning contact point A_f on the follower side of the tool, the elongation of the contact length in these points is assumed to be equal, because the clearance between the tooth of the tool and the pinion is geometrically the same.

Therefore, elongation $Ad_3Ad'_3$ at the beginning contact point and elongation $R_{f3}R_f{}'_3$ at the final contact point are equal to half of the difference of $Ad'_3R_f{}'_3$ measured value of the total length and Ad_3R_{f3} calculated value.

Judging from the strain of tooth root of consecutive contact teeth, the length elongated at points Rd_3 and A_{f3} are calculated as $Rd_3Rd'_3$ and $A_{f3}A_f{}'_3$. Therefore, the part of the elongation of contact length on the drive side of the tool is AdAd′ at the beginning contact point, and RdRd′ at the final contact point. In the follower side, the values are $A_fA_f{}'$ and $R_fR_f{}'$ respectively. These values are around 2 mm.

When the deformation of the tooth is not considered, the contact length on the line of action is 21.6 mm and the total length on the line is 38.6 mm. When the tooth is transformed in the rolling or the tool approaches the pinion, the elongation of the contact length becomes about 10% of the contact length at the beginning point and the final contact point. This value cannot be ignored.

4. Contact at Tooth Tip Considering Approach of Tool

4.1 Case in rack and pinion

The mechanism of elongation at the beginning contact point is shown in Fig. 4. In Fig. 4(a), the drive side of tool is at point Ad, and it contacts with the pinion tooth, and the tooth profile of toll is at point $A_d{}'$ on the line of action, it is shown by a broken line. It is shown to magnify the interval $A_dA_d{}'$ in Fig. 4(b). R_k in the figure is the radius of the outside diameter of the pinion. It does not change, even if this is rolled. O is the center of the gear, and P is the pitch point. The straight line which pass through P is parallel to the pitch line of the basic rack and it is called the mating pitch line. R_g is the radius of the base circle, α_c is the pressure angle, and δh is the amount of approach on the center distance for the tool.

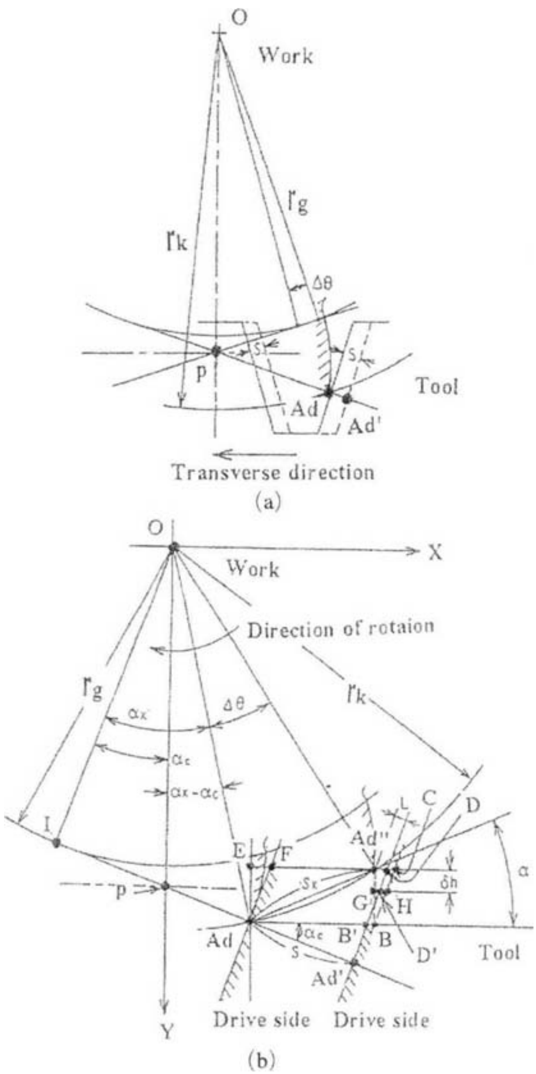

Fig. 4 Space between teeth at just before approach contact rack and pinion

When the rack tooth is at position A_d', the pinion tooth is located at position A_d'' in the right side $\triangle\theta = S/R_g$ from position A_d. At this position, it is possible to show the clearance L on the line of action between the tooth tip of the pinion tooth A_d'' and the tool tooth. However, R_k, α_k, α_q, θ_s change by the number of teeth and A.M. factor of the pinion.

$$
\begin{aligned}
L(\delta h=0) &= A_d''D\cdot\cos\alpha_c \\
&= (EF+FD-A_dB)\cos\alpha_c \qquad (2) \\
&= S((R_k/R_g)(\sin\alpha\tan\alpha_c-\cos\alpha)+\sec\alpha_c)\cos\alpha_c
\end{aligned}
$$

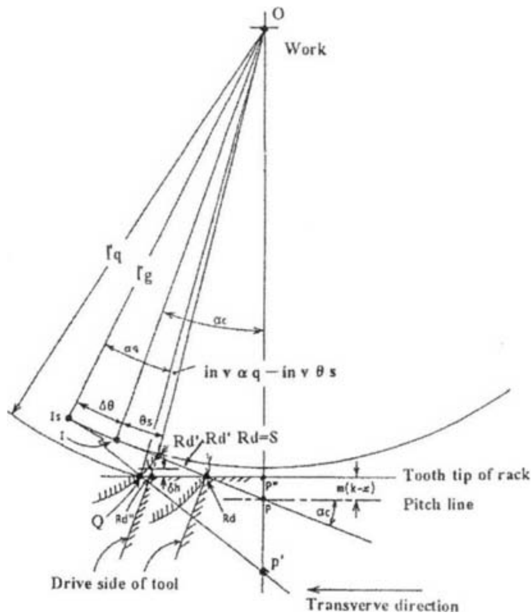

Fig. 5 Space between teeth at just out of recess contact rack and pinion

However, α is the angle between the straight line which connects with point A_d and A_d'' and a pitch line of the rack, and it is given by the equation :

$$
\tan\alpha = \frac{\cos(\alpha k+\Delta\theta-\alpha c)-\cos(\alpha k-\alpha c)}{\sin(\alpha k+\Delta\theta-\alpha c)-\sin(\alpha k-\alpha c)} \qquad (3)
$$

When the rolling force was applied to the tool, the tooth tip of the pinion is displaced from point A_d'' to point G, and the clearance L between teeth on the line of action will be as follows :

$$
\begin{aligned}
L(\delta h>0) &= GD'\cdot\cos\alpha_c \\
&= (A_d''D-\delta h\cdot\tan\alpha_c)\cos\alpha_c \qquad (4) \\
&= L(\delta h=0)-\delta h\cdot\sin\alpha_c
\end{aligned}
$$

The elongation mechanism at the final contact point Rd between the drive side of tool and the driven side of the pinion tooth is shown in Fig 5. The tooth profile of the tool transfers from the final contact point Rd to point Rd″. By converting the distance into the line of action, the result is S. In this interval, the pinion rotates the minute angle $\triangle\theta = S/R_g$. In the meantime, the intersection between the straight line R_dR_d'' described by the tool tip and the pinion tooth is shown by Q.

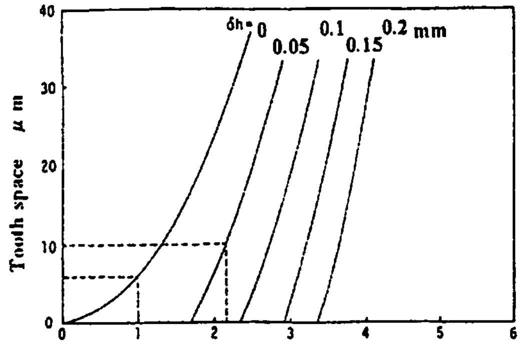

Fig. 6(a) Space between pinion tip and tooth root of rack

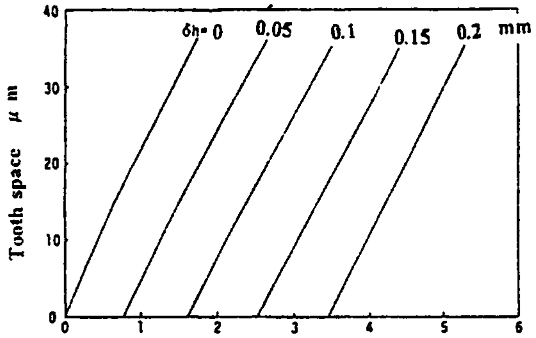

Fig. 6(b) Space between tooth tip of tool and pinion root

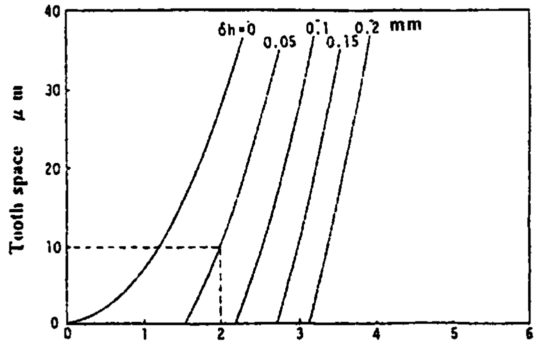

Fig. 7(a) Space between pinion tip and tooth root of rack

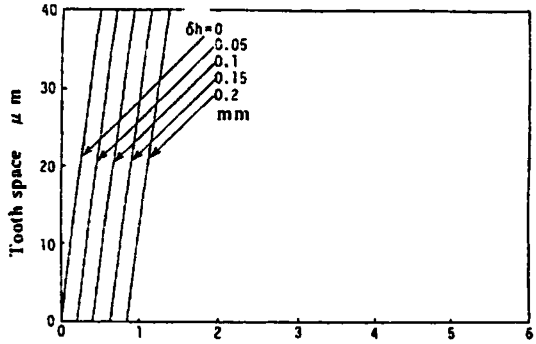

Fig. 7(b) Space between tooth tip of tool and pinion root

When the rolling force is not applied to the tool then the clearance between the tooth profile of the tool and the pinion tooth using symbol θ_s, α_q, R_q is shown by the following equation.

$$L(\delta h=0)=QRd''\cdot\cos\alpha_c = R_q(\text{inv}\,\alpha_q-\text{inv}\,\theta_s)\cdot\cos\alpha_c \quad (5)$$

When the rolling force was applied to the tool, the tooth tip approaches the pinion tooth by δh.

$$L(\delta h>0)=L(\delta h=0)-\delta h\cdot\sin\alpha_c \quad (6)$$

where, $\theta s=\tan^{-1}((IRd-S))/R_g)$

$\alpha_q=\tan^{-1}(IsQ/R_g.)$

$Rq=R_g\sec\alpha_q$

As a result, the clearance between the teeth is calculated in the mating gear set.

The relationship between the clearance of teeth and the distance S from the beginning contact point or the final contact point are shown in Fig. 6 for module 5, number of teeth 22, A.M. factor X=0. The results of the beginning contact point are shown in Fig. 6(a), and of final contact point in Fig. 6(b).

If at the beginning contact point, $\delta h=0$ and distance S=1 mm from A_d point, the clearance is 6 μm. If this clearance is filled with deflection of tooth or tooth profile error, the contact length elongates 1 mm. When the tooth tip approaches the pinion by $\delta h=0.05$ mm, and if the sum of the deflection of the tooth and the tooth profile error is 10 μm, the contact length elongates by 2.1 mm. Because the clearance at the final contact point is wider than the beginning contact point, the elongation of the contact length become shorter.

The relationship between the clearance of tooth and distance S from the beginning contact point or the final one is shown in Fig. 7 by module 5, number of teeth 22, A.M. factor X=0.52. The results of the beginning contact point are shown in Fig. 7(a), and those of the final contact point in Fig. 7(b).

The clearance between the teeth is affected by A.M. factor. When A.M. factor increases, the clearance also becomes wider and the elongation of contact length does not appear.

4.2 Contact at tooth in gear set

The mechanism of elongation at the beginning contact point Ad is shown in Fig. 8. The center of the tool is at O_1, and the center of pinion is at O_2. The line of action in the drive side of tool is shown by I_1I_2.

When the tooth profile of the tool is at point C, that is closer for point I_1 than the beginning contact point A_d, the tooth profile of the tool is shown by the broken line passing through C. The tooth tip of the workpiece is at point B which is in the right side $\triangle\theta_2$ from point A_d as shown in the figure. The tangent that passes through point B is drawn on the base circle of the tool. The contact point is I_1'. Therefore, when the tooth profile is at point C, the clearance between the teeth becomes B B′. Here, R_{g1} is the base circle radius of the tool, α is the mating pressure angle, and $L(=O_1O_2)$ is the center distance.

Other symbols are shown in the figure.

$$\begin{aligned} BB' &= BI_1' - B'I_1' = BI_1' - (I_1I_1' + CI_1) \\ &= R_{g1}\tan\phi_{ac} - R_{g1}(\phi_{a1} + \phi_{ac} - \alpha) - (A_dI_1 - A_dC) \end{aligned} \quad (7)$$

Where,

$$\phi_{a1} = \tan^{-1}(Rk2 \cdot \sin\phi_{a2}/(L - Rk_2\cos\phi_{a2}))$$
$$\phi_{ac} = \tan^{-1}(BI_1'/R_{g1})$$

When the tool approaches the pinion by δh, the center distance becomes $L-\delta h$. In the figure, it is not shown that the tool approaches the pinion. In this case, the tooth tip B moves by δh in parallel direction to the center line O_1O_2. The tangent passes through point B, and is drawn in contact with the base circle of the tool. The clearance between the tooth profile is the distance between point B and the tooth profile on this line, and it is calculated in the same way as $\delta h=0$.

The clearance at the tooth tip for module 5, in a standard gear of 44 and 22 teeth is shown in

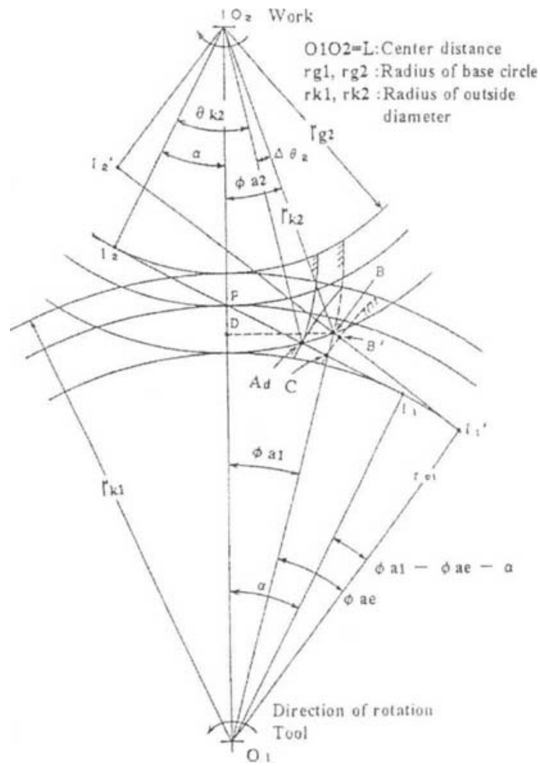

Fig. 8 Space between teeth at just before approach contact gear and pinion

Fig 9(a). If the amount of approach is $\delta h=0$, the clearance is 0 μm at point A_d. If the distance from point A_d is $S=1$ mm, the clearance becomes 13 μm. If it is $\delta h=0.1$ mm, the clearance becomes 0 μm at $S=1.6$ mm, and becomes 15 μm at $S=2$ mm.

The clearance at tooth tip in standard gear of 44 teeth, and 22 teeth, A.M. factor 0.52, module 5, is shown in Fig. 9(b). In the case of the profile-shifted gear, the clearance between the tooth profile is slightly wider than the standard gears. Therefore, the elongation of the contact length becomes shorter in the positive profile-shifted gear.

In the case of the gear set, thc clearance between teeth is considerably wide in the rack and pinion. Therefore, the elongation of the contact length in the pinion type tool is shorter than in the rack type tool.

In the case of $\delta h=0$, the clearance in the beginning contact point and the final contact

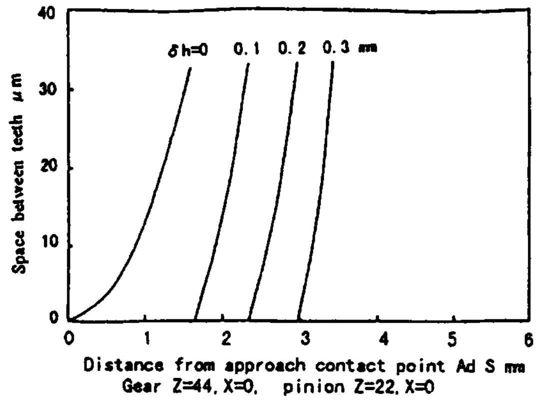

Fig. 9(a) Space between tooth tip of pinion and tooth root of gear

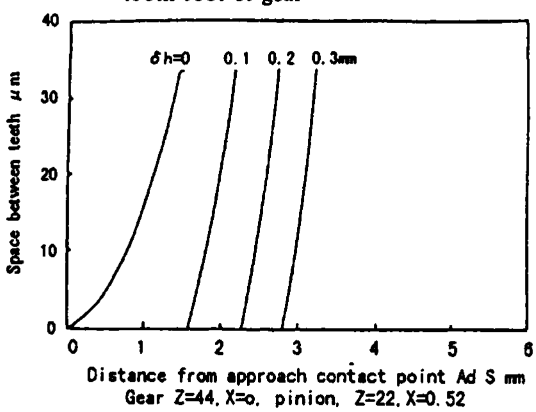

Fig. 9(b) Space between tooth tip of gear and tooth root of pinion

point when the center distance is constant. The method explained here incloudes the result of Akiyama et al. (1968) and Seager (1976) when δ h=0. However, in the case of Akiyama et al. (1968) and Seager (1976), the calculation formula is discussed approximately at the tooth tip contact and the calculated result is different.

5. Conclusion

The elongation of the contact length in the load single flank mating is comparatively short, as the tool does not approach the pinion with respect to the center distance. However, it can not be neglected in the rolling process. The elongation of the contact length becomes about 20% of the calculated kinematic value, and this value can not be disregarded in the tooth profile analysis.

The elongating interval is slightly before the beginning contact point and after the final contact point. If the clearance in this interval is filled, by the deflection of pinion tooth, tooth profile error, and the axial approach of the gear to the pinion, the contact length elongates in this interval.

The theory expounded here can also explain the phenomenon of the general case of single flank mating in which the gear shafts do not approach.

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